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Electron–hole recombination via a simplified cascade process

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Abstract. The normal Shockley–Read process of electron–hole recombination is generalised by allowing capture into an effective excited state of a trap which represents all its excited states. The familiar cascade capture is thus truncated and converted into an electron–hole recombination model which may be called truncated cascade recombination. The model differs from earlier work by the present authors in allowing direct capture also into the ground state, a process which is important at elevated temperatures. The earlier work considered also excited hole states, but these are omitted here for reasons of simplicity. The new model is discussed with a view to interpreting the dependence of emission and capture rates on temperature and the dependence of the steady-state recombination lifetime on injection.

1. Introduction

The present work has its origin in the need to discuss the electric field dependence of emission and capture rates of current carriers. This little-understood area is discussed by us in an independent companion paper (Landsberg and Dhariwal 1988), whose full implications depend on the theory developed here. It is desirable to split the argument in this way in order that the electric field effects can be discussed more or less independently of the more statistical aspects, which are the subject of this paper. Both papers deal with the same model, but for further applications to effects other than the electric field dependences the present paper is needed.

The cascade model has been known for many years and assumes that electrons cascade down a ladder of excited states to reach the ground state of a trap if they are not re-emitted at one of the intermediate stages. This involves the complication of multiphonon processes and even the statistics of these processes is involved. Simpler approaches have been proposed. For a review, see Stoneham (1975).

In recent years, we have developed a model which curtails the cascade to a single excited state for electron capture but adds an excited state also for hole capture. By also adding the valence band to the model, we have converted the theory from a simplified

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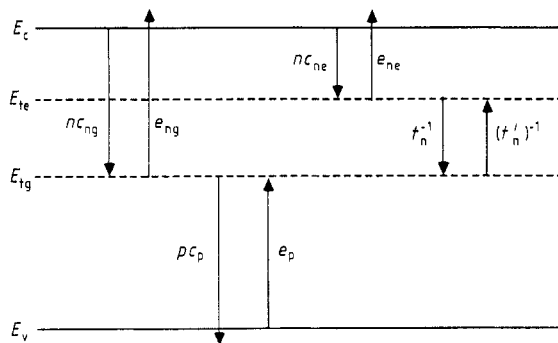


Figure 1. Transition rates used in this paper.

cascade capture theory to a generalised Shockley–Read–Hall (SRH) recombination theory (Dhariwal *et al* 1981, Landsberg and Abrahams 1983). Figure 1 gives a schematic description of the model. Although a certain amount of work had been done on cascade capture theories up to the 1980s, this conversion of it to an electron–hole recombination theory was an essentially new step. It did not, however, allow for the possibility of a direct transition between the ground state of the trap and the bands, as it was still too much influenced by the cascade philosophy. In this paper, this process is added (figure 1). This is not a physically trivial step since direct electron capture into the ground state can become an important mechanism at elevated temperatures. The mechanism of capture is left uninvestigated but may be presumed to be multi-phonon in nature. For simplicity, we have omitted the excited state for hole capture from consideration. We have called our model ‘truncated cascade recombination’ (TCR).

Some specific relevant papers on simplified cascades include those by Rzhano (1962) and Guro and Rzhano (1963), papers cited by Stoneham (1975) and a more recent paper by Gibb *et al* (1977). These theories do not include either the valence band or direct capture into the ground state. Also they deal mainly with the steady state whereas we also discuss the transient state in this paper. To help readers, table 1 supplies a comparison of various notation.

2. The kinetics of the model

In the present model of TCR, ν_0 , ν_g and ν_e , respectively, are the concentration of empty traps (e.g. ionised donors), the concentration of traps with electrons in their ground state and the concentration of traps with electrons in an excited state. The capture probability per unit time of a conduction band electron into the donor ground state is taken to be $C_{ng}(E)N(E) dE$. Here the electron is assumed to come from an energy range $(E, E + dE)$ for which the density of states per unit volume is $N(E) dE$. The emission probability per unit time in the reverse direction is denoted by $C'_{ng}(E)N(E) dE$. Both probabilities presume the donor centre empty or occupied in its ground state so that the actual net transition rate per unit volume from conduction band to trap ground state is

$$R_{ng} = \nu_0 nc_{ng} - \nu_g e_{ng}. \quad (2.1)$$

Table 1. Comparison with the results and notation used in earlier work.

Present work	Landsberg and Abrahams (1983)	Dhariwal <i>et al</i> (1981)	Gibb <i>et al</i> (1977)	Rzhanov (1962), Guro and Rzhanov (1963)
C_{ng}	—	—	—	—
C_{ne}	G	c_n	$\sigma_c v_{th}$	c_n
C_p	H	c_p	—	c_p
t_n	t_n	δt_n	ν^{-1}	r_n^{-1}
t'_n	t'_n	$\delta t'_n$	$[\nu \exp(-E_2/kT)]^{-1}$	—
—	t_p	δt_p	—	r_p^{-1}
—	t'_p	$\delta t'_p$	—	—
n_g	ν_1	n_1	$N \exp(-E_1/kT)$	n_1
n_c	n_2	$N_c \exp(-\Delta E_n/kT)$	$N \exp(-E_2/kT)$	n_1^*
p_g	π_1	p_1	—	p_1
τ_{nl}	—	$(N_1 c_n)^{-1} + [(N_c \delta t_n)/N_1] \times \exp(-\Delta E_n/kT)$	—	$(c_n N_c)^{-1} + n_1^*/N_1 r_n$
U_{max}	$N_i/(t_n + t_p)$	$N_i/(\delta t_n + \delta t_p)$	—	N_i/τ_r
$\eta_c - \eta_{tg}$	$\eta_c - \eta_t$	$(E_c - E_t)/kT$	$(E_1 + E_2)/kT$	$(E_c - E_t^*)/kT$
$\eta_c - \eta_{te}$	$\eta_c - (\eta_t + \delta)$	$\Delta E_n/kT$	E_1/kT	$(E_c - E_t)/kT$

With the conduction band Fermi function denoted by f_n , two transition rates averaged over the conduction band have been defined:

$$nc_{ng} \equiv \int_{CB} C_{ng}(E)N(E)f_n(E) dE \tag{2.2}$$

$$(S_c - n)c'_{ng} \equiv \int_{CB} C'_{ng}(E)N(E)[1 - f_n(E)] dE \equiv e_{ng} \tag{2.3}$$

where $S_c - n$ is the concentration of unoccupied conduction band states. Here e_{ng} is the conventionally used 'emission rate' from the trap in its ground state. For the excited states, one can write similarly after integrating over the conduction band

$$R_{ne} = \nu_0 nc_{ne} - \nu_e e_{ne}. \tag{2.4}$$

The assumed isotropy of the distribution function limits the validity of the theory to low fields.

The equilibrium concentrations are

$$(\nu_0, \nu_g, \nu_e)_{eq} = (Z_0, (\exp \gamma_{eq})Z_{1g}, (\exp \gamma_{eq})Z_{1e}) N_T/[Z_0 + (\exp \gamma_{eq})(Z_{1g} + Z_{1e})] \tag{2.5}$$

where γ_{eq} is the equilibrium Fermi level divided by kT , Z_0 is the partition function of the empty trap and $Z_1 = Z_{1g} + Z_{1e}$ is the partition function for the filled trap, split into ground-state and excited-state terms. By detailed balance for an energy range dE and writing $\eta = E/kT$

$$[C'_{ng}(E)/C_{ng}(E)]_{eq} = [\{f_n(E)/[1 - f_n(E)]\}(\nu_0/\nu_g)]_{eq} = (Z_0/Z_{1g}) \exp(-\eta) = C'_{ng}(E)/C_{ng}(E). \tag{2.6}$$

The last step assumes excitation independence (Landsberg 1979) of the capture coefficients. Using (2.3), (2.6) and

$$\exp(-\eta)[1 - f_n(E)] = \exp(-\gamma_n)f_n(E) \quad (2.7)$$

where γ_n is the quasi-Fermi level (divided by kT) for the conduction band,

$$e_{ng} = \frac{Z_0}{Z_{1g}} \int \exp(-\eta)C_{ng}(E)[1 - f_n(E)]N(E) dE = \frac{Z_0}{Z_{1g}} \exp(-\gamma_n)nc_{ng}. \quad (2.8)$$

Let us generalise (2.5) by the use of quasi-Fermi levels $kT\gamma_g$ and $kT\gamma_e$ for the ground and excited states of the trap

$$(\nu_0, \nu_g, \nu_e) = (Z_0, (\exp \gamma_g)Z_{1g}, (\exp \gamma_e)Z_{1e}) \\ \times N_T/[Z_0 + (\exp \gamma_g)Z_{1g} + (\exp \gamma_e)Z_{1e}]. \quad (2.9)$$

Inserting (2.8) and (2.9) into (2.1) gives

$$R_{ng} = [1 - \exp(\gamma_g - \gamma_n)]\nu_0nc_{ng}. \quad (2.10)$$

In thermal equilibrium, $R_{ng} = 0$. A similar argument shows that (2.4) yields

$$R_{ne} = [1 - \exp(\gamma_e - \gamma_n)]\nu_0nc_{ng}. \quad (2.11)$$

Next, the electron transition rate per unit volume from the ground state to the valence band is

$$R_{gp} = \nu_g pc_p - \nu_0 e_p.$$

Here c_p is an average, corresponding to (2.2), but now over the valence band:

$$pc_p \equiv \int_{VB} C_p(E)[1 - f_n(E)]N(E) dE \\ (S_v - p)c'_p \equiv \int_{VB} C'_p(E)f_n(E)N(E) dE \equiv e_p.$$

By detailed balance and assuming excitation independence,

$$[C'_p(E)/C_p(E)]_{eq} = (Z_{1g}/Z_0) \exp \eta = C'_p(E)/C_p(E)$$

so that

$$e_p = (Z_{1g}/Z_0)(\exp \gamma_p)pc_p \quad (2.12)$$

and

$$R_{gp} = [1 - \exp(\gamma_p - \gamma_g)]\nu_g pc_p \quad (2.13)$$

where γ_p is the quasi-Fermi level (divided by kT) for the valence band.

The net transition rate per unit volume from the excited states to the ground state of a trap will be written in the form

$$R_{eg} = \nu_e/t_n - \nu_g/t'_n \quad (2.14)$$

where by detailed balance

$$(t'_n/t_n)_{eq} = (\nu_g/\nu_e)_{eq} = Z_{1g}/Z_{1e}. \quad (2.15)$$

It is convenient to define certain new concentrations as ratio of averages of the type (2.2) and (2.3):

$$n_g \equiv e_{ng}/c_{ng} = (Z_0/Z_{1g})n \exp(-\gamma_n) \quad n_e \equiv e_{ne}/c_{ne} = (Z_0/Z_{1e})n \exp(-\gamma_n) \quad (2.16)$$

$$p_g \equiv e_p/c_p = (Z_{1g}/Z_0)p \exp \gamma_p. \quad (2.17)$$

These relations, together with others in this section, hold also if the bands are Fermi degenerate. If they are, the concentrations (2.16) and (2.17) depend on the Fermi levels. They correspond to relations (15.13) in Landsberg (1982). In the non-degenerate case the new concentrations are Fermi level independent and correspond to the usual Shockley-Read parameters (η_T is the trap energy level divided by kT)

$$n_1 \equiv n \exp(\eta_T - \gamma_e) \quad p_1 \equiv p \exp(\gamma_p - \eta_T). \quad (2.18)$$

The latter can be obtained from (2.16) and (2.17) by introducing effective trap levels of energy $E_{Tg} = kT\eta_{Tg}$ and $E_{Te} = kT\eta_{Te}$ by

$$Z_0/Z_{1g} = \exp \eta_{Tg} \quad Z_0/Z_{1e} = \exp \eta_{Te}. \quad (2.19)$$

If several excited states exist, E_{Te} must be expected to be temperature dependent.

The net recombination rates per unit volume can now also be written in the form

$$R_{ng} = (\nu_0 n - \nu_g n_g) c_{ng} \quad R_{ne} = (\nu_0 n - \nu_e n_e) c_{ne} \quad (2.20)$$

$$R_{gp} = (\nu_g p - \nu_0 p_g) c_p. \quad (2.21)$$

It should also be noted that the quantity c_{ng} of relation (2.2) need not describe only a single-electron transition given by a recombination coefficient $T_1^S(E)$, say, to use a common notation (Landsberg 1982, p 405). Trap Auger effects involving the conduction band (T_1) or the valence band (T_2) for the second electron can be regarded as included. In that case, one has to interpret

$$c_{ng} \text{ as } T_{1g}^S + T_{1g} n + T_{2g} p. \quad (2.22)$$

Similarly, one can interpret

$$c_p \text{ as } T_2^S + T_3 n + T_4 p \quad (2.23)$$

$$c_{ne} \text{ as } T_{1e}^S + T_{1e} n + T_{2e} p. \quad (2.24)$$

3. The transient properties of the concentrations

If one eliminates the concentration $\nu_0 (= N_t - \nu_g - \nu_e)$, one finds, as the differential equations of the model, the coupled pair

$$\dot{\nu}_e = R_{ne} - R_{eg} = a - b\nu_e - c\nu_g \quad (3.1)$$

$$\dot{\nu}_g = R_{eg} + R_{ng} - R_{gp} = d - h\nu_e - l\nu_g. \quad (3.2)$$

The constants are given in table 2 later in this section. The above equations have to be solved together with the electron and hole continuity equations

$$\dot{n} = -(R_{ne} + R_{ng}) + G + (1/q)\nabla \cdot \mathbf{j}_n \quad (3.3)$$

$$\dot{p} = -R_{gp} + G - (1/q)\nabla \cdot \mathbf{j}_p \quad (3.4)$$

where \mathbf{j}_n and \mathbf{j}_p , respectively, are the electron and hole current densities and G is the generation rate for electrons and holes. However, for experimental conditions in which n and p are kept time independent, one finds that

$$\dot{\nu}_i + P\nu_i + Q\nu_i + S_i = 0 \quad (i \equiv 0, g, e). \quad (3.5)$$

Similar equations have been obtained by Rees *et al* (1980) by neglecting electron transitions to the ground state and hole transitions and by Pickin (1978) for the electron-donor recombination.

Table 2. Expressions for constants.

Defining equation	Constant	Dimension	Expression for constant
(3.1)	$\begin{cases} a \\ b \\ c \end{cases}$	$\begin{matrix} L^{-3}T^{-1} \\ T^{-1} \\ T^{-1} \end{matrix}$	$\begin{matrix} nc_{ne}N_i \\ nc_{ne} + e_{ne} + 1/t_n \\ nc_{ne} - 1/t'_n \end{matrix}$
(3.2)	$\begin{cases} d \\ h \\ l \end{cases}$	$\begin{matrix} L^{-3}T^{-1} \\ T^{-1} \\ T^{-1} \end{matrix}$	$\begin{matrix} (nc_{ng} + e_p)N_i \\ nc_{ng} + e_p - 1/t_n \\ nc_{ng} + e_{ng} + pc_p + e_p + 1/t'_n \end{matrix}$
(3.5)	P	T^{-1}	$b + l = n(c_{ng} + c_{ne}) + pc_p + e_{ng} + e_{ne} + e_p + 1/t_n + 1/t'_n$ (3.9)
(3.5)	Q	T^{-2}	$bl - ch$ $= c_{ne}c_pnp + [(n_e + n_g)c_{ng}c_{ne} + (c_{ng} + c_{ne})(1/t_n + 1/t'_n)]n$ $+ (n_e c_{ne} + 1/t_n)c_p p$ $+ (n_g c_{ng} + p_g c_p)(n_e c_{ne} + 1/t_n) + (c_{ne} + p_g c_p/n_e)(n_g/t_n)$ (3.10)
(3.5)	S_g	$L^{-3}T^{-2}$	$ah - bd$ $= -(nc_{ng} + p_g c_p)(t_n^{-1} + n_e c_{ne})N_i - nc_{ne}N_i t_n^{-1}$ (3.11)
(3.5)	S_e	$L^{-3}T^{-2}$	$cd - al$ $= -(n_g c_{ng} + pc_p)nc_{ne}N_i - (nc_{ng} + p_g c_p + nc_{ne})N_i/t'_n$ (3.12)
(3.5)	S_0	$L^{-3}T^{-2}$	$-S_e - S_g - QN_i$ (3.13)

If one uses

$$\mu_i \equiv \nu_i + S_i/Q$$

then

$$\ddot{\mu}_i + P\dot{\mu}_i + Q\mu_i = 0.$$

This is a homogeneous linear differential equation with constant coefficients so that

$$\mu_i(t) = \nu_i(t) + S_i/Q = A_i \exp(-\omega_1 t) + B_i \exp(-\omega_2 t)$$

where

$$\omega_1 \equiv \tau_1^{-1} = (P/2)[1 + (1 - 4Q/P^2)^{1/2}] \quad \omega_2 \equiv \tau_2^{-1} = (P/2)[1 - (1 - 4Q/P^2)^{1/2}]. \quad (3.6)$$

Inserting some initial conditions by assuming $\nu_i(0)$ and $\dot{\nu}_i(0)$ to be given and noting that, for positive ω_1 and ω_2 ,

$$\nu_i(\infty) = -S_i/Q \quad (i \equiv 0, g, e) \quad (3.7)$$

one finds the transient solutions for $i \equiv 0, g, e$:

$$\begin{aligned} (\omega_1 - \omega_2)[\nu_i(t) - \nu_i(\infty)] &= \{\dot{\nu}_i(0) + \omega_1[\nu_i(0) - \nu_i(\infty)]\} \exp(-\omega_2 t) \\ &- \{\dot{\nu}_i(0) + \omega_2[\nu_i(0) - \nu_i(\infty)]\} \exp(-\omega_1 t). \end{aligned} \quad (3.8)$$

The explicit and exact expressions for τ_1 , τ_2 , and $\nu_i(\infty)$ can be obtained from table 2 using (3.1) and (3.7).

For our numerical work we use the parameters given in table 3. The capture cross section σ_{ne} has been assumed to be of the order of the largest measured cross section (Bonch-Bruevich and Landsberg 1968) and more typical values have been chosen for σ_{ng} and σ_p . They have been assumed to be independent of temperature. The resulting effective electron capture cross section, which is defined below in (3.25), does show a temperature dependence, however. The excited states have been assumed to have an

Table 3. Values of parameters used in the present work and parameters inferred from them at $T = 300$ K.

Parameters used in present work	
σ_{nc}	10^{-13} cm^2
$\sigma_{ng} = \sigma_p$	10^{-16} cm^2
v_{th} (for electrons and holes)	$10^7 (T/300)^{1/2} \text{ cm s}^{-1}$
N_t	10^{13} cm^{-3}
$E_c - E_{Tc}$	0.05 eV
$E_c - E_{Tg} = E_{Tg} - E_v$	0.55 eV
t_n	10^{-10} s
$N_c = N_v$ (at 300 K)	$2 \times 10^{19} \text{ cm}^{-3}$
Parameters inferred from the above parameters	
$n_e = N_c \exp[-(\eta_c - \eta_{Tc})]$	$2.89 \times 10^{18} \text{ cm}^{-3}$
$n_g = N_c \exp[-(\eta_c - \eta_{Tg})]$	$1.15 \times 10^{10} \text{ cm}^{-3}$
$p_g = N_v \exp[-(\eta_{Tg} - \eta_v)]$	$1.15 \times 10^{10} \text{ cm}^{-3}$
$t'_n = t_n n_e / n_g$	0.0251 s

Table 4. Steady-state occupation probabilities $f_i = \nu_i / N_T$ for a depleted semiconductor and under a uniform injection of electron concentration.

Condition	T (K)	p (cm^{-3})	n (cm^{-3})	f_g	f_e	f_0
Depletion	300	0	0	0.18357	2.5178×10^{-12}	0.81602
	200	0	0	0.04532	2.2861×10^{-16}	0.95467
	100	0	0	0.00166	6.2767×10^{-29}	0.99833
Uniform injection of electrons	300	0	10^{12}	0.98865	3.9232×10^{-9}	1.1340×10^{-2}
	200	0	10^{12}	0.99999	2.5094×10^{-13}	1.5010×10^{-7}
	100	0	10^{12}	1.00000	6.2973×10^{-26}	0.0000

effective energy level at 50 meV below the conduction band edge. This is in accordance with the arguments given by Lax (1960), Abakumov *et al* (1977b) and others who assume that electrons are captured by levels with binding energy greater than kT . As to traps, Rees *et al* (1980) have obtained from the measured emission rates (Grimmeiss *et al* 1980) for singly charged S and Se centres in Si a value of $t_n \approx 0.5 \times 10^{-10}$ s. We have kept to the same order of magnitude here.

In table 4, typical values of the steady-state occupation probabilities $f_i = \nu_i(\infty) / N_T$ are given for two areas of practical interest. In the first case the semiconductive layer is depleted of carriers ($p = n = 0$), e.g. by reverse biasing a p-n-junction, whereas in the second case a uniform flux of electrons is injected in the otherwise depleted semiconductor. In either case, because n is small, f_e is very small. Thus, excited states, although important for the capture of electrons, do not hold them (as will be seen later, such an assumption is violated under a high injection condition). Usually, for a depleted semiconductor, most of the traps are empty. Thus $f_0 \approx 1$, whereas $f_g \ll 1$. These conditions are best met at low temperatures. Similarly, for uniform electron injection, all the traps are filled with electrons, giving $f_g \approx 1$ and $f_0 \ll 1$.

Table 5. Calculated values of parameter involved in a transient for measurement of emission rate ($n = p = 0$), for the semiconductor parameters given in table 3.

T (K)	P (s^{-1})	Q (s^{-2})	$\tau_1 \equiv \omega_1^{-1}$ (s)	$\tau_2 \equiv \omega_2^{-1}$ (s)
300	2.90×10^{12}	1.81×10^{14}	3.44×10^{-13}	1.59×10^{-2}
260	1.62×10^{12}	4.33×10^{12}	6.16×10^{-13}	3.74×10^{-1}
220	7.79×10^{11}	3.11×10^{10}	1.28×10^{-12}	2.49×10^1

Table 6. Calculated values of parameters involved in a transient for the measurement of capture rate ($p = 0, n = 10^{12} \text{ cm}^{-3}$) for the semiconductor parameters given in table 3.

T (K)	P (s^{-1})	Q (s^{-2})	$\tau_1 = \omega_1^{-1}$ (s)	$\tau_2 = \omega_2^{-1}$ (s)
240	1.15×10^{12}	9.97×10^{15}	8.69×10^{-13}	1.15×10^{-4}
180	2.96×10^{11}	7.97×10^{15}	3.37×10^{-12}	3.71×10^{-4}
120	3.50×10^{10}	6.34×10^{15}	2.82×10^{-11}	5.57×10^{-6}

Two types of experiment usually performed to determine the emission and capture rates correspond to switching the semiconductor from one to the other of the above said conditions (Sah *et al* 1970). In the first set of experiments, traps (donors here) are first filled with electrons and then, at some time $t = 0$, electrons and holes are swept out of the conduction and valence bands by applying a large reverse bias. In the depletion layer, we then have $n = p \approx 0$ at $t > 0$. Therefore the condition of constant n and p is met and (3.8) applies.

Under constant reverse bias, decay of $f_g(t)$ from near unity to a small value occurs as electrons are emitted from the traps. The values of the time constants are given in tables 5 and 6 using the values of the parameters given in table 3. It is seen that

$$\tau_1 (\equiv \omega_1^{-1}) \ll \tau_2 (\equiv \omega_2^{-1}) \quad 4Q/P^2 \ll 1. \tag{3.14}$$

Hence, from (3.6),

$$\tau_1^{-1} \approx P \quad \tau_2^{-1} \approx Q/P. \tag{3.15}$$

Consider now the emission experiment, denoted by a suffix e attached to the time constants, and assume that $n_e c_{ne} \gg n_g c_{ng}, p_g c_p$. Then an approximate relation for the readjustment time τ_1 of the population of the excited states is obtained from (3.9) by noting that only the last five terms contribute and, of these, the terms e_{ne} and $1/t_n$ dominate. Hence

$$\tau_1 \rightarrow \tau_{1e} = t_n / (1 + n_e c_{ne} t_n). \tag{3.16}$$

Similarly, for τ_{2e} , we have

$$\tau_{2e}^{-1} = (Q/P)_{n=p=0} = [c_{ng} + c_{ne} / (1 + n_e c_{ne} t_n)] n_g + c_p p_g = e_{neff} + e_p \tag{3.17}$$

where

$$e_{neff} \equiv [c_{ng} + c_{ne} / (1 + n_e c_{ne} t_n)] n_g \tag{3.18}$$

is an effective emission rate for electrons.

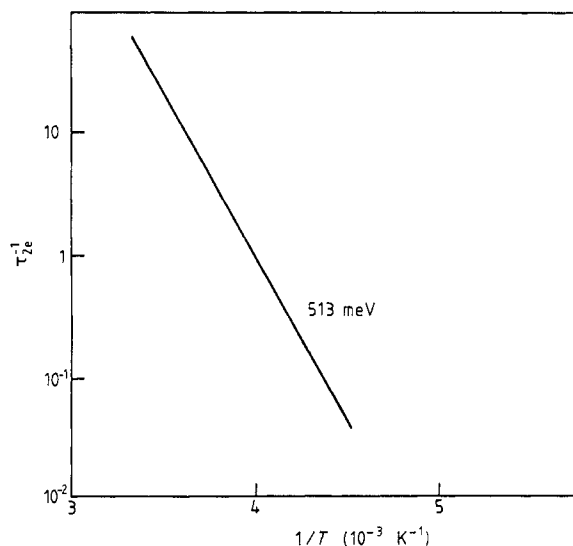


Figure 2. A plot of τ_{2e}^{-1} against $1/T$ according to (3.17) using parameters of table 3.

For table 5, $\tau_{2e} \gg \tau_{1e}$, so that after a short initial time the transient is dominated by τ_{2e} and, from (3.8),

$$\nu(t) - \nu(\infty) = [\nu(0) - \nu(\infty)] \exp(-t/\tau_{2e}) \quad (3.19)$$

where $\nu \equiv \nu_g + \nu_e$ is the concentration of trapped electrons. In a transient capacitance experiment, the above equation is used to determine τ_{2e} , whose variation with temperature gives the activation energy of the trap. From (3.17), we find that τ_{2e} involves three different activation energies which cannot always be distinguished experimentally. The assumption that only one activation energy dominates, which was made by Gibb *et al* (1977), can be derived if the following approximations are made:

$$e_p \ll e_{neff} \quad c_{ng} \ll c_{ne}/(1 + n_e c_{ne} t_n) \quad 1 \ll n_e c_{ne} t_n.$$

Then

$$\tau_{2e}^{-1} \approx n_g/t_n n_e = (1/t_n) \exp[-(\eta_{Te} - \eta_{Tg})]. \quad (3.20)$$

Although the interpretation based on this relation appears justified in the case of Gibb *et al*, the present analysis shows its limitations. In particular, the last approximation becomes poorer at lower temperatures.

In figure 2, we have plotted τ_{2e}^{-1} against $1/T$ for these parameters. The curve shown is only approximately straight with an activation energy of 513 meV. In the approximation (3.20) the straight line is exact with an activation energy of 500 meV.

The second set of transient experiments involving ν_i is for direct measurement of the effective capture rate of electrons at empty traps, usually by using a reverse-biased junction. At $t \leq 0$, we assume that $\nu_0 = N_T$. Since for $t \leq 0$ we have a steady state, we can write $\dot{\nu}_i(0) = 0$. Now, at $t = 0$, electron-hole pairs are injected, say by light. Electrons, which are minority carriers, flow steadily into the depletion layer with a drift velocity v_{dn} such that $\Delta n = j_n/v_{dn}$ is constant with respect to time. Thus, in the transient phase, both n and p are again constant and (3.8) applies. Values of the time constants

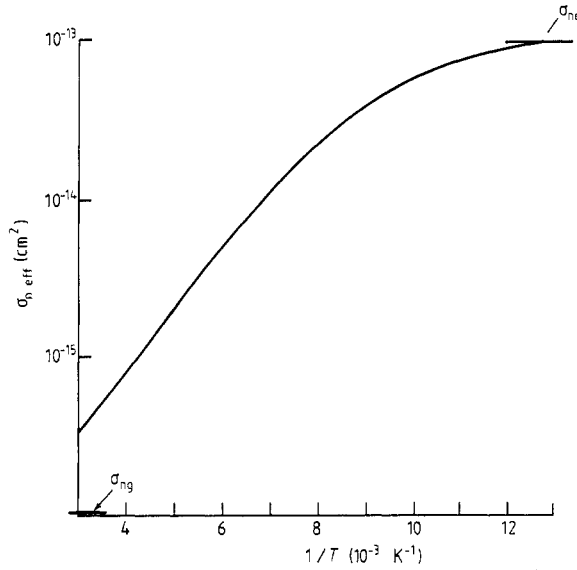


Figure 3. A plot for the present theory of $\sigma_{n,eff}$ against $1/T$ showing that the cross sections for electron capture into the ground state and into the effective excited state represent its limiting values. The parameters of table 3 have been used.

for this case are given in table 6 and show that relations (3.14) and (3.15) hold. The low-injection capture time constants can therefore be defined by

$$\tau_{1c}^{-1} \equiv (P)_{\substack{p=0 \\ n=\Delta n \ll n_c}} \approx \tau_{1c}^{-1}$$

and

$$\tau_{2c}^{-1} = (Q/P)_{p=0, \Delta n \ll n_c} = [c_{ng} + c_{ne}/(1 + n_e c_{ne} t_n)] \Delta n + e_{neff} + e_p. \tag{3.21}$$

A subscript c has been used with τ to denote a capture experiment. Also note that at low temperatures the first term on the right-hand side of (3.21) dominates. Since it clearly deals only with capture it can be used to define an effective capture coefficient,

$$c_{neff} \equiv (1/\tau_{2c} \Delta n)_{low\ temp} = c_{ng} + c_{ne}/(1 + n_e c_{ne} t_n). \tag{3.22}$$

By (3.18) and (3.22),

$$e_{neff}/c_{neff} = n_g = (\tau_{2c} \Delta n/\tau_{2e})_{low\ temp}. \tag{3.23}$$

In capture experiments the trapped electron concentration behaves as

$$\nu(t) = \nu(\infty)[1 - \exp(-t/\tau_{2c})] \quad (t \gg \tau_{1c}). \tag{3.24}$$

An effective electron capture cross section is

$$\sigma_{neff} \equiv c_{neff}/v_{th}. \tag{3.25}$$

In figure 3, we have plotted σ_{neff} against $1/T$ and find that, at low temperatures, σ_{neff} is close to σ_{ne} whereas, at high temperatures, it tends to σ_{ng} . Thus, direct capture into the ground state, even if not of interest at low temperatures, may become significant at room temperature.

Equation (3.22) can be viewed as an approximation to a cascade theory (Lax 1959,

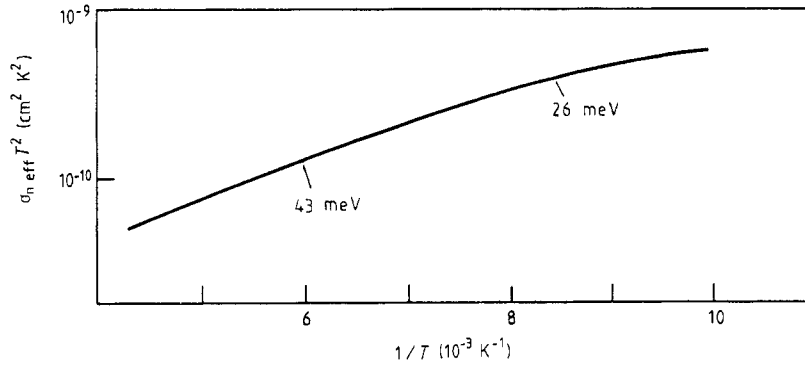


Figure 4. A plot for the present theory of $\sigma_{\text{neff}}T^2$ against $1/T$ based on (3.22) and (3.25) showing that one does not find a straight line. The parameters of table 3 have been used and two activation energies are shown which apply for a small range of temperatures.

1960, Lal and Landsberg 1965, Smith and Landsberg 1966) (see Stoneham (1975) for a review) in which all the excited states have been lumped together into a single effective energy level E_{T_e} defined by (2.19). Sticking probabilities P_i that an electron from a state i will reach the ground state, can be identified by writing (3.22) as

$$c_{\text{neff}} = c_{\text{ng}} + c_{\text{ne}}P_e$$

whence

$$P_e = 1/(1 + n_e c_{\text{ne}} t_n) = t_n^{-1}/(t_n^{-1} + e_{\text{ne}}). \tag{3.26}$$

This shows that the present approach is consistent with the cascade theory, simplified to have one effective excited level. Equations (3.22) and (3.26) have been obtained by a simple steady-state argument in our companion paper (Landsberg and Dhariwal 1988).

If one neglects direct transitions to the ground state ($c_{\text{ng}} = 0$) and assumes that $n_e c_{\text{ne}} t_n \gg 1$, one arrives at

$$\tau_{2c}^{-1} = \Delta n/n_e t_n$$

as used by Gibb *et al* (1977, equation (14)). They also obtain (3.23) (Gibb *et al* 1977, equation (11)) even though their individual expressions (denoted by ϵ_T and $\sigma_T v_{\text{th}}$ in their paper) refer to their special case. By these assumptions Gibb *et al* (1977) and Rees *et al* (1980) get $\sigma_{\text{neff}} \approx (v_{\text{th}} n_e t_n)^{-1}$. Since $v_{\text{th}} \sim T^{1/2}$ and $n_e \sim T^{3/2} \exp(\eta_{T_e} - \eta_c)$ a plot of $\log(\sigma_{\text{neff}} T^2)$ against T^{-1} should give a straight line. For the present theory, one finds figure 4 which it is difficult to approximate by a straight line. If such an approximation is made for a limited range of temperatures, one is liable to obtain a misleading value of the activation energy. Gibb *et al* have determined $E_c - E_{T_g}$ by calculating $e_{\text{neff}}/c_{\text{neff}}$ using (3.23) and also by calculating the sum of the energy $E_{T_c} - E_{T_g}$ obtained from equivalent of our figure 2 and the energy $E_c - E_{T_e}$ obtained from the equivalent of figure 4. For the same set of experimental data, they obtain different values by these two methods. In view of our comments on figures 2 and 4, their evaluation based on their figures is bound to be unreliable and only (3.23) should be used. This would resolve the difficulty noted in their paper. These experiments, however, are performed for the depletion layer, where a large electric field exists. Effective capture and emission rates are modified by the presence of an electric field (Abakumov *et al* 1977a, Rosier and Sah 1971) as discussed by Landsberg and Dhariwal (1988).

Table 7. Parameters in (4.5).

$$\begin{aligned}
\alpha &= c_{ne}c_p t_n \\
\beta &= c_{ng}(1 + n_g c_{ne} t_n + n_e c_{ne} t_n + t_n/t'_n) + c_{ne}(1 + t_n/t'_n) \\
\gamma &= c_p(1 + n_e c_{ne} t_n) \\
\delta &= n_g c_{ng} + p_g c_p + n_g n_e c_{ng} c_{ne} t_n + n_e p_g c_{ne} c_p t_n + (t_n/t'_n)(n_e c_{ne} + p_g c_p) \\
\lambda &= c_{ng}(1 + n_e c_{ne} t_n) + c_{ne} \\
\mu &= c_p(1 + n_e c_{ne} t_n + c_{ne}/c_{ng}) \\
\rho &= n_g c_{ng}(1 + n_e c_{ne} t_n) + p_g c_p(1 + n_e c_{ne} t_n) + (c_{ne}/c_{ng})(n_g c_{ng} + p_g c_p)
\end{aligned}$$

4. The steady-state recombination rate

On substituting the expressions $\nu_i(\infty)$ in a net recombination rate formula such as (2.21), one finds the steady-state recombination rate U . The expression normally found in SRH statistics will be denoted by U_{SRH} . One then finds that

$$U_{SRH} = np[1 - \exp(\gamma_p - \gamma_n)]c_{ng}c_p N_T / [(n + n_g)c_{ng} + (p + p_g)c_p] \quad (4.1)$$

$$U = [(1 + n_e c_{ne} t_n + c_{ne}/c_{ng}) / (1 + n_e c_{ne} t_n + A)] U_{SRH} \equiv F U_{SRH} \quad (4.2)$$

where

$$\begin{aligned}
A &\equiv \{[n_e c_{ne}(p c_p t_n + n_g c_{ng} t_n + 1) \\
&\quad + (t_n/t'_n)[(n + n_e)c_{ne} + n c_{ng} + p_g c_p]] / [(n + n_g)c_{ng} + (p + p_g)c_p]\}.
\end{aligned} \quad (4.3)$$

Note that one can put $c_{ne} \approx t_n \approx 0$ on neglecting the excited states, whence $U \rightarrow U_{SRH}$.

Now, for any electron lifetime, one has

$$\tau_n = \Delta n / U = \Delta n / F U_{SRH} = (1/F) \tau_{SRH}. \quad (4.4)$$

This turns out to be the product of three factors

$$\begin{aligned}
\tau_n &= [(\alpha np + \beta n + \gamma p + \delta) / (\lambda n + \mu p + \rho)] [(n_e c_{ng} + p c_p + n_g c_{ng} + p_g c_p) / p c_{ng} c_p N_T] \\
&\quad \times [\Delta n / n \{1 - \exp[-(\gamma_n - \gamma_p)]\}].
\end{aligned} \quad (4.5)$$

The first factor is F^{-1} , and the last factor is usually of order unity and is part of τ_{SRH} . The new parameters (table 7) α , β , γ , δ , λ , μ and ρ depend on c_{ng} , c_{ne} , c_p , t_n , t'_n , n_g , n_e and c_p and can readily be obtained from (4.1)–(4.3). If the last factor of (4.5) is put equal to unity, one finds for the low level ($p \gg n$, n_g , P_g) and high injection level ($\Delta n \approx n \approx p \gg n_g$, p_g) lifetimes, respectively,

$$\begin{aligned}
\tau_{nl} &= \gamma / \mu c_{ng} N_T = (1 + n_e c_{ne} t_n) / [c_{ng}(1 + n_e c_{ne} t_n) + c_{ne}] (1/N_T) \\
&= 1/c_{neff} N_T \rightarrow 1/c_{ng} N_T
\end{aligned} \quad (4.6)$$

$$\tau_{nh} = \{[\alpha^2 n^2 + (\beta + \gamma)n] / (\lambda + \mu)n\} [(c_{ng} + c_p)n / p c_{ng} c_p N_T] \quad (4.7)$$

$$\begin{aligned}
&= [n c_{ne} c_p t_n + (1 + n_e c_{ne} t_n)(c_{ng} + c_p) + c_{ne}(n_g c_{ng} t_n + 1 + t_n/t'_n) \\
&\quad + c_{ng} t_n/t'_n] / [(1 + n_e c_{ne} t_n)c_{ng} + c_{ne}] c_p N_T \rightarrow (c_{ng} + c_p) / c_{ng} c_p N_T.
\end{aligned} \quad (4.8)$$

Note that the c_{neff} of the transient problem occurs again here in the steady-state case. The arrows indicate the results in the limit in which one goes to Shockley–Read statistics. In (4.7), one need not retain the term $(\beta + \gamma)n$ for very large n , but without it one would obtain zero in the Shockley–Read limit of (4.8). One also has

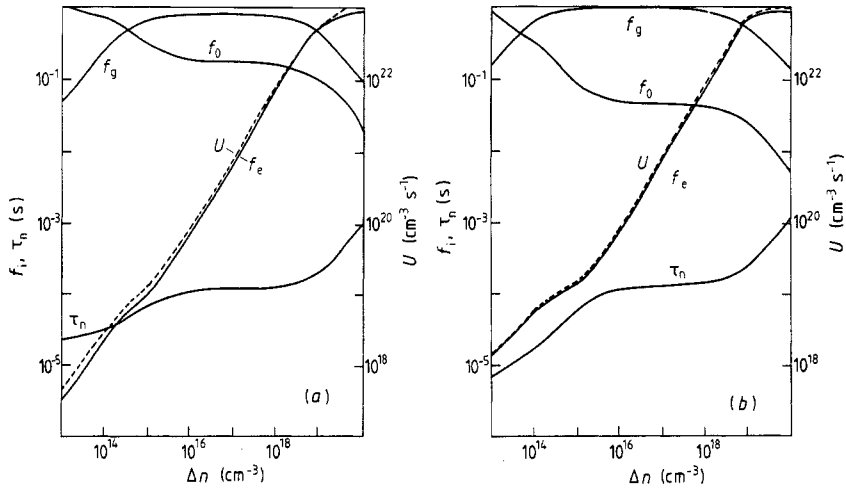


Figure 5. Steady state occupation probabilities f_i , minority-carrier lifetime τ_n and steady-state recombination rate U per unit volume as functions of excess minority carrier concentration n at (a) $T = 300$ K and (b) $T = 200$ K. Other parameters are given in table 3.

$$\tau_{nh} = \tau_{nl} + (1 + t_n/t'_n)/c_p N_T + n c_{ne} t_n / [(1 + n_e c_{ne} t_n) c_{ng} + c_{ne}] N_T$$

$$\rightarrow 1/c_{ng} N_T + 1/c_p N_T. \tag{4.9}$$

Thus, τ_{nh} consists of three terms. The first deals with electron capture and dominates in p-type material at low injection. The second term is a contribution due to hole capture, which is for many cases of practical interest (in which $t_n/t'_n \ll 1$) the same as in the SRH theory. The last term is new and shows that there can be a significant increase in the lifetime at high injection. This is due to the bottleneck produced at the excited levels whose delay time t_n leads to the fact that they are largely occupied, thus withdrawing their availability for electron capture. In this last stage of a high injection,

$$\tau_{nh} \approx (n t_n / N_T) / [1 + (c_{ng}/c_{ne})(1 + n_e c_{ne} t_n)]. \tag{4.10}$$

The proportionality of τ and $n = \Delta n$ implies that at this level the steady-state recombination rate $U = \Delta n / \tau_n$ reaches a constant plateau as a function of Δn .

To understand the effect arising owing to the level of injection, we have calculated the fractions

$$f_i = \nu_i / N_T \quad i \equiv 0, g, e \tag{4.11}$$

for empty traps and for traps filled with electrons in the ground and excited states, respectively. These have been plotted against Δn in figure 5. The semiconductor has been assumed to be of p-type Si with $p_0 = 10^{15} \text{ cm}^{-3}$.

At a low level of injection ($\Delta n \approx 10^{13} \text{ cm}^{-3}$), f_0 is close to unity whereas f_g and f_e are very small. Thus the deciding factor for recombination is the minority-carrier capture (into the excited or ground state of the trap). The lifetime of excess electron-hole pairs is then given by (4.6). As Δn is increased, more and more traps are filled, thereby reducing the concentration $\nu_0 = f_0 N_T$ of the traps available for electron capture. At a high level of injection ($10^{15} \text{ cm}^{-3} < \Delta n < 10^{18} \text{ cm}^{-3}$) the recombination is mostly determined by the hole capture ($f_g > f_0$) and the lifetime becomes nearly constant with respect to Δn . In figure 5, we have plotted f_i at two different temperatures and find that the ratio f_g/f_0 is higher at lower temperatures. In the high-level range,

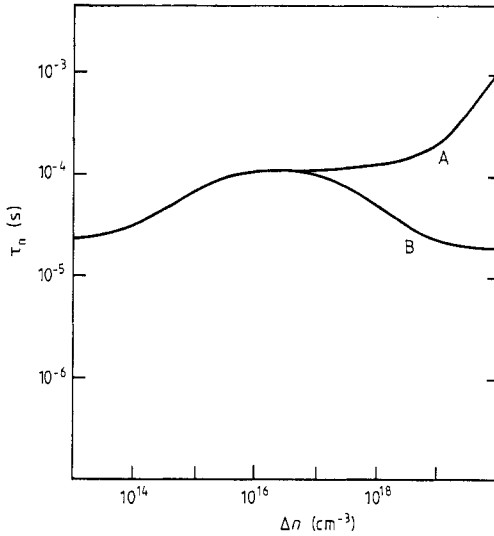


Figure 6. Curve A shows the lifetime curve of figure 5(a). Curve B includes Auger recombination coefficients $T_{1g} = T_{2g} = T_3 = T_4 = 10^{-27} \text{ cm}^6 \text{ s}^{-1}$ and $T_{1e} = T_{2e} = 10^{-24} \text{ cm}^6 \text{ s}^{-1}$. An effect of Auger recombination is to reduce the minority-carrier lifetime at high injections.

although f_e is very small, it keeps on increasing almost proportionally to Δn . Since $f_0 + f_g + f_e = 1$, this increase in f_e is at the cost of f_0 and f_g and they start to decrease. As shown in figure 5, U follows f_e very closely, the small difference being due to direct capture into the ground state. An approximate value of U is given by

$$U = R_{ng} + R_{eg} = f_0 N_t c_{ng} \Delta n + N_t f_e / t_n \tag{4.12}$$

where the reverse process of thermal emission has been neglected at high levels of injection. As Δn increases, $f_e \rightarrow 1$ and $f_0 \rightarrow 0$ and a maximum value of U may be approximated as (Dhariwal *et al* 1981, Landsberg and Abrahams 1983)

$$U_{\Delta n \rightarrow \infty} \approx N_t / t_n.$$

However, the more accurate expression (4.2) for U has a limiting value

$$U_{\Delta n \rightarrow \infty} = (N_t / t_n) [1 + (c_{ng} / c_{ne}) (1 + n_e c_{ne} t_n)]. \tag{4.13}$$

Such a saturation of U has been reported in the literature (Fabre *et al* 1975, Pogany 1980, figure 6). It is interesting to note that, even when a direct transition into the ground state has been allowed, the trap saturation occurs because of the readjustment of the electron population between the ground state and the excited states.

Because U approaches a constant in (4.13), the lifetime (4.4) increases in proportion to Δn at high levels of injection. Such an effect, however, may not be observable in some cases because of other recombination processes. For example, if the Auger coefficients in (2.22) and (2.23) became important, the lifetime may decrease and then become constant. This has been shown in figure 6.

We note in conclusion that additional experimental studies on the dependence of τ_n on Δn exist and tend to give somewhat conflicting result (Ashkinadze *et al* 1972, Dalal and Moore 1977, Glinchuk *et al* 1972, Ho and Mathias 1983, Ho *et al* 1977, Ivanov

1966, Spirito and Cocorullo 1987, Suemune *et al* 1987, Wilson 1967, Zimmerman 1973). Further studies are therefore required.

5. Generation lifetime

Some remarks have to be added about the rate of generation of thermal carriers in the steady state. The condition corresponds typically to a reverse-biased p-n junction and the resulting current is called the reverse saturation current. In the absence of electrons and holes ($n = p = 0$), equation (4.2) gives the rate of generation

$$G = -(U)_{n=p=0} = (1 + n_e c_{ne} t_n + c_{ne}/c_{ng}) c_{ng} c_p N_T n_i^2 / [(1 + n_e c_{ne} t_n) (n_g c_{ng} + p_g c_p) + (t_n/t'_n) (n_e c_{ne} + p_g c_p)] = N_t e_{neff} e_p / (e_{neff} + e_p) \quad (5.1)$$

where e_{neff} has been defined in (3.18).

A generation lifetime (Schroder 1982, Rawlings 1987) is defined as

$$\tau_g \equiv n_i / G = (n_i / N_t) (1/e_p + 1/e_{neff}). \quad (5.2)$$

Thus in the steady state the generation rates e_p and e_{neff} form two paths *in series*, and the slower of the two decides the rate of generation. This is in contrast with the time constant τ_{2e} (equation (3.17)) involved in the transient emission experiment which is due to a *parallel* combination of e_p and e_{neff} . Thus, measurements of τ_g and τ_{2e} give complementary information. This is not the case with τ_{nl} (equation (4.6)) and τ_{2c} (equation (3.21)). They are related by

$$c_{neff} = 1/\Delta n \quad \tau_{2c} = 1/N_t \tau_{nl}.$$

6. Concluding remarks

A simplified cascade electron-hole recombination model has been developed. It lumps all excited states of a recombination centre into one effective energy level. The study of *decay rates* leads to short-time constants τ_1 and long-time constants τ_2 for capture as well as emission rates (tables 5 and 6). Effective emission and capture rates have been defined and their temperature dependences have been analysed. This analysis enables one to remove some anomalies in the identification of activation energies from emission and capture experiments (§ 3). The quantities which govern the steady-state recombination rate are algebraically involved, but simplifying assumptions have been shown to make them quite transparent. The effective capture coefficient of the decay theory is again important and a variety of behaviour of lifetime as a function of injection is predicted, some of which have been observed (§ 4).

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